Homework 5, due 3/27

1. Let $\mathcal{U} = \{U_0, U_1\}$ denote the usual open cover of $\mathbf{P}^1$. Compute the cohomology group $\check{H}^1(\mathcal{U}, \mathcal{O}(-1))$.

2. Let X be a complex manifold, and $Y \subset X$ a codimension-one complex submanifold.

   (a) Show that X has a cover with open sets U_α, such that there are holomorphic functions $f_\alpha \in \mathcal{O}(U_\alpha)$ satisfying $Y \cap U_\alpha = f_\alpha^{-1}(0)$, and in addition we can choose the f_α such that the differential df_α does not vanish along $Y \cap U_\alpha$. In this case for any α, β we have $f_\alpha/f_\beta : U_\alpha \cap U_\beta \to \mathbf{C}^*$.

   (b) Define a line bundle L over X by the transition functions $\phi_{\alpha\beta} = f_\alpha/f_\beta$ on $U_\alpha \cap U_\beta$. Show that L admits a global holomorphic section s such that $Y = s^{-1}(0)$.

   (c) Show that the normal bundle $\mathcal{N}_{Y|X}$ of Y in X is isomorphic to the restriction of the bundle L to Y.